

Aligned Interference Neutralisation for $2 \times 2 \times 2$ Interference Channel with Imperfect Channel State Information

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Abstract—We investigate the efficiency of interference alignment for $2 \times 2 \times 2$ interference networks formed by concatenation of two user interference channels under the assumption of imperfect channel state information. We show that, as a result of cumulative noise in zero forcing receivers, aligned interference neutralisation is not practical when perfect channel state information is not available. We give analytical SNR and SINR expressions at relays and receivers and also provide some simulation results using a channel estimation error model.

I. INTRODUCTION

The interference channel is a good model for many networks such as cellular networks, wireless local area networks, wireless ad-hoc networks where multiple transmitter and receiver pairs use the same radio frequencies. This arrangement results in co-channel interference. In these networks, it is well known that the performance of each user is interference limited.

Interference alignment (IA) is a recent concept where the undesired signal component (interference) is aligned to one half of signal dimensions and leaving the other half to the desired signal [1]. In a network consisting of two users (two transmitter-receiver pair), the principles of IA will enable the users to communicate interference-free for half of the time. Extending this to K users, in order to avoid interference each user should be able to communicate interference free for $1/K$ of the time using TDMA [1]. With interference alignment, we can provide interference-free communication to the K users now for $1/2$ of the time. In the parlance of IA, we can say that the degree of freedom for the 2 user interference channel is 1 [2].

Authors of [2] have extended the IA scheme to K user MIMO interference networks where transmitters and receivers have the same number of antennas. In [3], a general case of unequal number of antennas at all transmitters and receivers is studied and innerbound/outerbound for the total number of degrees of freedom is provided. In [4], IA is extended to cellular networks, especially for more than two-cell cases where there are multiple non-intended BSs, as subspace interference alignment. The authors of [5] applied IA to a two-cell interfering two user MIMO-MAC network with a new feedback framework which results in an improvement over

random vector quantization feedback in [6], [7].

IA with relays was first considered in [8] for a MIMO Y Channel. In this system, 3 users, each convey and receive independent messages to/from the other 2 users via a relay. Authors of [9] have studied a SISO network consisting of two sources, two relays and two destinations, where the first hop is between the sources (transmitters) and the relays and the second hop is between the relays and the destinations (receivers). This setting, which is very appealing as it has been shown to provide 2 degrees of freedom for 2 user networks; can be considered as a concatenation of 2 interference channels. In this paper, the authors propose aligned interference neutralisation as a way to align the interference over each hop of the network leading the interference to be cancelled out over the last hop. This achieves the 2 degrees of freedom. However, the remarkable benefit of aligned interference neutralisation has been shown under idealized assumptions, i.e. the availability of perfect channel knowledge which is not realistic in wireless networks.

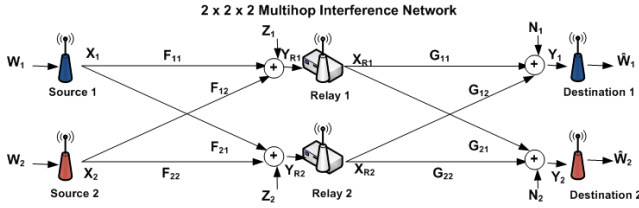
In this paper, we analyse the effects of imperfect channel state information (CSI) on the performance of aligned interference neutralisation for a 2 transmitter - 2 relay - 2 receiver channel of [9] as shown in Figure 1a. This is referred to as a $2 \times 2 \times 2$ network and its information theoretic capacity is of interest. Channel estimation errors cause erroneous beamforming vectors and thus misalignment of desired and interference signals as shown in Figure 1b. Our specific contributions are as follows.

- We derive analytical expressions for the SNR and SINR at the relays. For the former we show that, due to the zero forcing at the relays, the expected value of the noise tends to infinity. Similarly the interference caused by misalignment due to CSI errors is shown to be very prominent with the mean tending to infinity.
- We demonstrate that the SINR analysis for the relays can be extended to the destinations and show that there are cumulative effects of interference and noise at both stages.
- We validate the analysis through system simulations. We show that the interference due to CSI errors can be modelled via simulation by a log normal distribution, thus

confirming the presence of prominent instantaneous values of interference.

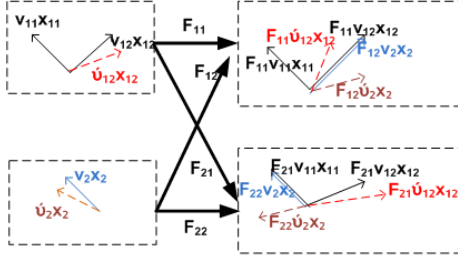
This paper is organized as follows. The system model, aligned interference neutralisation in $2 \times 2 \times 2$ interference channel is described in Section II. The performance analysis giving SNR and SINR expressions are introduced in Section III. Section IV presents simulation results including those for the modelling the interference statistics. Conclusions are given in Section V.

II. SYSTEM MODEL

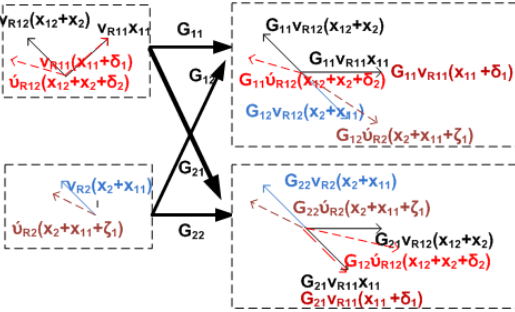


(a) 2 User 2 Transmitter 2 Relay 2 Receiver ($2 \times 2 \times 2$) Network

Alignment in the First Hop with Perfect and Imperfect CSI



Alignment in the Second Hop with Perfect and Imperfect CSI



(b) Interference Alignment with perfect / imperfect beamforming vectors

Fig. 1. System Model

A. Time varying Channel Coefficients and Perfect CSI

We consider the time varying channel coefficient case in [9] where the transmission occurs over M time slots. Transmitters 1 and 2 encode M and $M - 1$ data symbols x_{1,k_1} , x_{2,k_2} respectively, using beamforming vectors \mathbf{v}_{1,k_1} and \mathbf{v}_{2,k_2} as shown in the first hop of Figure 1b, resulting in $(M \times 1)$ data

vectors

$$\mathbf{X}_1 = \sum_{k_1=1}^M \mathbf{v}_{1,k_1} x_{1,k_1} \quad (1)$$

$$\mathbf{X}_2 = \sum_{k_2=1}^{M-1} \mathbf{v}_{2,k_2} x_{2,k_2}. \quad (2)$$

In the case of perfect CSI, the beamforming vectors are designed to align the interference at the relays [9] and are given by

$$\mathbf{v}_{1,i+1} = (\mathbf{F}_{11}^{-1} \mathbf{F}_{12} \mathbf{F}_{22}^{-1} \mathbf{F}_{21})^i \mathbf{v}_{1,1} \quad (3)$$

$$\mathbf{v}_{2,i} = (\mathbf{F}_{22}^{-1} \mathbf{F}_{21} \mathbf{F}_{11}^{-1} \mathbf{F}_{12})^{i-1} \mathbf{F}_{22}^{-1} \mathbf{F}_{21} \mathbf{v}_{1,1}, \quad (4)$$

where $\mathbf{v}_{1,1}$ is arbitrarily set to $\mathbf{v}_{1,1} = [1, 1, \dots, 1]^T$, $\forall i \in \{1, \dots, M-1\}$. Let the M symbol extension of the link from Transmitter k to Relay j be denoted by a diagonal matrix $\mathbf{F}_{jk}(t) = \text{diag}\{f_{jk}(Mt+1), \dots, f_{jk}(Mt+M)\}$. Similarly the links between relay j and receiver n are given by $\mathbf{G}_{nj}(t) = \text{diag}\{g_{nj}(Mt+1), \dots, g_{nj}(Mt+M)\}$. Then the $(M \times 1)$ signal vectors received at relay R_j and the receiver n where $j = 1, 2$ and $n = 1, 2$, are:

$$\mathbf{Y}_{R_j}(t) = \mathbf{F}_{j1}(t) \mathbf{X}_1(t) + \mathbf{F}_{j2}(t) \mathbf{X}_2(t) + \mathbf{Z}_j(t) \quad (5)$$

$$\mathbf{Y}_n(t) = \mathbf{G}_{n1}(t) \mathbf{X}_{R_1}(t) + \mathbf{G}_{n2}(t) \mathbf{X}_{R_2}(t) + \mathbf{N}_n(t). \quad (6)$$

Omitting the time dependence of channel coefficients for notation simplicity, the alignment conditions result in:

$$\mathbf{Y}_{R_1} = \mathbf{F}_{11} \mathbf{X}_1 + \mathbf{F}_{12} \mathbf{X}_2 + \mathbf{Z}_1 \quad (7)$$

$$= \mathbf{F}_{11} \mathbf{v}_{1,1} x_{1,1} + \sum_{i=1}^{M-1} \mathbf{F}_{11} \mathbf{v}_{1,i+1} (x_{1,i+1} + x_{2,i}) + \mathbf{Z}_1 \quad (8)$$

and similarly;

$$\mathbf{Y}_{R_2} = \mathbf{F}_{21} \mathbf{X}_1 + \mathbf{F}_{22} \mathbf{X}_2 + \mathbf{Z}_2 \quad (9)$$

$$= \sum_{i=1}^{M-1} \mathbf{F}_{21} \mathbf{v}_{1,i} (x_{1,i} + x_{2,i}) + \mathbf{F}_{21} \mathbf{v}_{1,M} x_{1,M} + \mathbf{Z}_2, \quad (10)$$

where \mathbf{Z}_1 and \mathbf{Z}_2 are the $(M \times 1)$ noise vectors at the relays.

Interference is neutralised by applying zero forcing (ZF) at the relays, giving the $(M \times 1)$ received vectors

$$\mathbf{X}_{R_1} = \begin{bmatrix} x_{R_1,1} \\ x_{R_1,2} \\ \vdots \\ x_{R_1,M} \end{bmatrix} = \mathbf{F}_{R_1}^{-1} \mathbf{Y}_{R_1} = \begin{bmatrix} x_{1,1} \\ x_{1,2} + x_{2,1} \\ \vdots \\ x_{1,M} + x_{2,M-1} \end{bmatrix} + \mathbf{F}_{R_1}^{-1} \mathbf{Z}_1 \quad (11)$$

$$\mathbf{X}_{R_2} = \begin{bmatrix} x_{R_2,1} \\ x_{R_2,2} \\ \vdots \\ x_{R_2,M} \end{bmatrix} = \mathbf{F}_{R_2}^{-1} \mathbf{Y}_{R_2} = \begin{bmatrix} x_{1,1} + x_{2,1} \\ x_{1,2} + x_{2,2} \\ \vdots \\ x_{1,M} \end{bmatrix} + \mathbf{F}_{R_2}^{-1} \mathbf{Z}_2 \quad (12)$$

where $\mathbf{F}_{R_j} = [\mathbf{F}_{j1}\mathbf{v}_{1,1} \ \mathbf{F}_{j1}\mathbf{v}_{1,2} \cdots \mathbf{F}_{jM}\mathbf{v}_{1,M}]$ is the $(M \times M)$ zero forcing matrix at relay R_j . The relays transmit the signals to the destinations along beamforming vectors $\mathbf{v}_{R_1,i+1}$ and $\mathbf{v}_{R_2,i}$ defined

$$\mathbf{v}_{R_1,i+1} = (\mathbf{G}_{11}^{-1}\mathbf{G}_{12}\mathbf{G}_{22}^{-1}\mathbf{G}_{21})^i \mathbf{v}_{R_1,1} \quad (13)$$

$$\mathbf{v}_{R_2,i} = -(\mathbf{G}_{22}^{-1}\mathbf{G}_{21}\mathbf{G}_{11}^{-1}\mathbf{G}_{12})^{i-1} \mathbf{G}_{22}^{-1}\mathbf{G}_{21}\mathbf{v}_{R_1,1} \quad (14)$$

where once again $\mathbf{v}_{R_1,1} = [1, 1, \dots, 1]^T$.

Following the beamforming vectors at the relays using (13) and (14), the received signals at destinations 1 and 2 can be shown to be given by

$$\mathbf{Y}_1 = \mathbf{G}_{11}\mathbf{X}_{R_1} + \mathbf{G}_{12}\mathbf{X}_{R_2} + \mathbf{N}_1 \quad (15)$$

$$\begin{aligned} &= \mathbf{G}_{11}\mathbf{v}_{R_1,1}(x_{1,1} + z'_{1,1}) \\ &+ \sum_{i=1}^{M-1} \mathbf{G}_{11}\mathbf{v}_{R_1,i+1}(x_{1,i+1} - x_{1,i} + z'_{1,i+1} - z'_{2,i}) + \mathbf{N}_1 \end{aligned} \quad (16)$$

and

$$\mathbf{Y}_2 = \mathbf{G}_{21}\mathbf{X}_{R_1} + \mathbf{G}_{22}\mathbf{X}_{R_2} + \mathbf{N}_2 \quad (17)$$

$$\begin{aligned} &= \mathbf{G}_{21}\mathbf{v}_{R_1,M}(x_{1,M} + x_{2,M-1} + z'_{1,M}) \\ &+ \sum_{i=1}^{M-1} \mathbf{G}_{22}\mathbf{v}_{R_2,i}(x_{2,i} - x_{2,i-1} - z'_{1,i} + z'_{2,i}) + \mathbf{N}_2, \end{aligned} \quad (18)$$

where z'_{1,k_1} and z'_{2,k_2} are the k_1 th and k_2 th element of $\mathbf{F}_{R_1}^{-1}\mathbf{Z}_1$ and $\mathbf{F}_{R_2}^{-1}\mathbf{Z}_2$ in (11) and (12) respectively. Finally each destination decodes the data symbols successively along each dimension as described in [9].

B. Imperfect CSI Model

In modelling CSI imperfections, we consider the effective misalignment of transmitter and relay beamforming vectors in (3), (4) and in (13), (14) relative to the CSI obtained at each relay and destination, respectively. Since the relay and destination will perform ZF on the CSI estimated locally, we can assume that both relays and destinations obtain perfect CSI, but the information fed back from receivers to relays and from relays to the transmitters has errors. Thus, IA is performed using erroneous channel matrices ($\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$). In this paper, we modelled the CSI imperfections as in [10]. Therefore:

$$\hat{\mathbf{F}} = \rho\mathbf{F} + \bar{\rho}\mathbf{\Xi} \quad (19)$$

and

$$\hat{\mathbf{G}} = \rho\mathbf{G} + \bar{\rho}\mathbf{\Xi} \quad (20)$$

where ρ , $0 < \rho < 1$ and $\bar{\rho} = \sqrt{1 - \rho^2}$, control the amount of CSI imperfection (i.e. $\rho = 1$ refers to perfect CSI) and $\mathbf{\Xi}$ is an $(M \times M)$ diagonal complex Gaussian matrix with zero mean and unit variance. It is shown in [11] and [12] that ρ can be used to determine the impact of several factors on imperfect CSI and can be a function of the length of the training sequence, SNR and Doppler frequency.

III. PERFORMANCE ANALYSIS FOR IMPERFECT CSI

We now derive analytical expressions of SNR for perfect CSI and SINR for imperfect CSI model in (19), for the first hop. Then we extend this to end-to-end SINR for imperfect CSI. For mathematical clarity, we consider the case of $M = 2$. Expressions for $M > 2$ are not considered but can be derived following similar methodology.

A. SNR for Perfect CSI

Consider the received vector at Relay 1 in (11), which for $M = 2$, we denote by $\tilde{\mathbf{x}} = [x_{R_1,1} \ x_{R_1,2}]^T$. Using (11), after straightforward manipulation, one can show that:

$$\begin{aligned} \mathbb{E}[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^\dagger] &= \begin{bmatrix} \mathbb{E}|x_{1,1}|^2 & 0 \\ 0 & \mathbb{E}|x_{1,2}|^2 + \mathbb{E}|x_{2,1}|^2 \end{bmatrix} \\ &+ \sigma^2 \mathbb{E}[\mathbf{F}_{R_1}^{-1}\mathbf{F}_{R_1}^{-1\dagger}] \end{aligned} \quad (21)$$

where σ^2 is the noise power. Using the definition of \mathbf{F}_{R_j} given in Section IIA, gives the instantaneous SNR for symbol $x_{1,1}$

$$\gamma_{\text{inst}x_{1,1}} = \frac{\mathbb{E}|x_{1,1}|^2}{\frac{\sigma^2}{|\Delta|^2} \Delta_1} \quad (22)$$

where Δ_1 and Δ given in terms of the instantaneous channel coefficients are:

$$\Delta_1 = |f_{12}(2)|^2 |f_{21}(2)|^2 |f_{22}(1)|^2 + |f_{12}(1)|^2 |f_{21}(1)|^2 |f_{22}(2)|^2 \quad (23)$$

and

$$\Delta = f_{11}(1)f_{22}(1)f_{12}(2)f_{21}(2) - f_{12}(1)f_{21}(1)f_{11}(2)f_{22}(2) \quad (24)$$

with $f_{jk}(l)$ denoting the l^{th} diagonal entry of \mathbf{F}_{jk} .

To investigate the properties of (22), we examine the expected value of its $\frac{\Delta_1}{|\Delta|^2}$ denoted by

$$T = \mathbb{E} \left[\frac{\Delta_1}{|\Delta|^2} \right]. \quad (25)$$

Taking the expectation over $f_{11}(1)$ and $f_{11}(2)$ while fixing the others, we obtain

$$T = \mathbb{E} \left[\mathbb{E} \left[\frac{A}{|Bf_{11}(1) + Cf_{11}(2)|^2} \right] \right] \quad (26)$$

where A , B , C are functions of f_{12} , f_{21} , f_{22} and the inner expectation is over $f_{11}(1)$ and $f_{11}(2)$. In (26)

$$Bf_{11}(1) + Cf_{11}(2) \sim \mathcal{CN}(0, |B|^2 P_{11} + |C|^2 P_{11}) \quad (27)$$

where $\mathbb{E}(|f_{11}(1)|^2) = \mathbb{E}(|f_{11}(2)|^2) = P_{11}$, $B = f_{22}(1)f_{12}(2)f_{21}(2)$ and $C = -f_{12}(1)f_{21}(1)f_{22}(2)$. Thus,

$$\begin{aligned} |\Delta|^2 &= |Bf_{11}(1) + Cf_{11}(2)|^2 \\ &= P_{11}(|B|^2 + |C|^2)X_{11} \end{aligned} \quad (28)$$

where X_{11} is a unit mean exponential variable. Using (28) and (26), we have:

$$T = \mathbb{E} \left[\frac{|B|^2 + |C|^2}{P_{11}(|B|^2 + |C|^2)X_{11}} \right] \quad (29)$$

$$= \mathbb{E} \left[\frac{1}{P_{11}X_{11}} \right] \rightarrow \infty \quad (30)$$

since $\mathbb{E}(1/X_{11}) = \int_0^\infty \frac{1}{x} e^{-x} dx \rightarrow \infty$.

The above analytical system shows that even when perfect CSI is assumed, the mean value of the noise power is infinite. Finally substituting (28) into (22), we have the instantaneous SNR given by

$$\gamma_{\text{inst}_{x_{1,1}}} = \frac{\mathbb{E}[|x_{1,1}|^2]}{(\frac{\sigma^2}{P_{11}X_{11}})} \quad (31)$$

which is an exponential random variable with mean $\mathbb{E}[|x_{1,1}|^2] \frac{P_{11}}{\sigma^2}$.

B. SINR for Imperfect CSI

We now analyze the SINR for the symbol $x_{1,1}$ at Relay 1 for CSI error modelled by (19). The received vector \mathbf{Y}_{R_1} is then given by

$$\mathbf{Y}_{R_1} = \mathbf{F}_{11}(\hat{\mathbf{v}}_{1,1}x_{1,1} + \hat{\mathbf{v}}_{1,2}x_{1,2}) + \mathbf{F}_{1,2}\hat{\mathbf{v}}_{2,1}x_{2,1} + \mathbf{Z}_1$$

where errored beamforming vectors are given by:

$$\hat{\mathbf{v}}_{1,1} = \mathbf{v}_{1,1} = [1, 1]^T \quad (32)$$

$$\hat{\mathbf{v}}_{2,1} = \hat{\mathbf{F}}_{22}^{-1} \hat{\mathbf{F}}_{21} \mathbf{v}_{1,1} \quad (33)$$

$$\hat{\mathbf{v}}_{1,2} = \hat{\mathbf{F}}_{11}^{-1} \hat{\mathbf{F}}_{12} \hat{\mathbf{F}}_{22}^{-1} \hat{\mathbf{F}}_{21} \mathbf{v}_{1,1}. \quad (34)$$

Using (19), we can express the inverse of $\hat{\mathbf{F}}_{jk}$ by

$$\hat{\mathbf{F}}_{jk}^{-1} = (\rho \mathbf{F}_{jk} + \bar{\rho} \mathbf{\Xi}_{jk})^{-1} \quad (35)$$

$$= (\rho \mathbf{F}_{jk} (\mathbf{I} + \frac{\bar{\rho}}{\rho} \mathbf{F}_{jk}^{-1} \mathbf{\Xi}_{jk}))^{-1} \quad (36)$$

$$\cong \frac{1}{\rho} \mathbf{F}_{jk}^{-1} - \frac{\bar{\rho}}{\rho^2} \mathbf{F}_{jk}^{-1} \mathbf{\Xi}_{jk} \mathbf{F}_{jk}^{-1} \quad (37)$$

where we have neglected the second order terms $(\frac{\bar{\rho}}{\rho})^2$ following (36). Note that substituting (37) into (3) and (4) gives the erroneous beamforming vectors. Using the computation of (33) and (34) and following similar approach to equation (21) in Section IIIA, one can show the received vector at Relay 1 as $\tilde{\mathbf{x}} = [\hat{x}_{R_1} \ \hat{x}_{R_2}]^T$, with imperfect CSI and

$$\mathbb{E}[\tilde{\mathbf{x}}\tilde{\mathbf{x}}^\dagger] = \begin{bmatrix} \mathbb{E}|x_{1,1}|^2 & 0 \\ 0 & \mathbb{E}|x_{1,2}|^2 + \mathbb{E}|x_{2,1}|^2 \end{bmatrix} + \mathbb{E}[\mathbf{F}_{R_1}^{-1} \mathbf{\Omega} \mathbf{\Omega}^\dagger \mathbf{F}_{R_1}^{-1}] \quad (38)$$

where $\mathbf{\Omega} = \mathbf{F}_{11} \mathbf{\Theta}_{12} \mathbf{v}_{1,1} x_{1,2} + \mathbf{F}_{12} \mathbf{\Theta}_{21} \mathbf{v}_{1,1} x_{2,1} + \mathbf{Z}_1$. Denote $\mathbf{\Theta}_{12}, \mathbf{\Theta}_{21}$ by

$$\mathbf{\Theta}_{21} = \mathbf{A} \mathbf{\Xi}_{21} + \mathbf{B} \mathbf{\Xi}_{22} \quad (39)$$

$$\mathbf{\Theta}_{12} = \mathbf{C} \mathbf{\Xi}_{11} + \mathbf{D} \mathbf{\Xi}_{12} + \mathbf{F} \mathbf{\Xi}_{22} + \mathbf{G} \mathbf{\Xi}_{21} \quad (40)$$

where

$$\mathbf{A} = \frac{\bar{\rho}}{\rho} \mathbf{F}_{22}^{-1} \quad \mathbf{B} = -\frac{\bar{\rho}}{\rho} \mathbf{F}_{22}^{-2} \mathbf{F}_{21} \quad (41)$$

$$\mathbf{C} = -\frac{\bar{\rho}}{\rho} \mathbf{F}_{11}^{-2} \mathbf{F}_{12} \mathbf{F}_{22}^{-1} \mathbf{F}_{21} \quad \mathbf{D} = \frac{\bar{\rho}}{\rho} \mathbf{F}_{11}^{-1} \mathbf{F}_{22}^{-1} \mathbf{F}_{21} \quad (42)$$

$$\mathbf{F} = -\frac{\bar{\rho}}{\rho} \mathbf{F}_{22}^{-2} \mathbf{F}_{11}^{-1} \mathbf{F}_{12} \mathbf{F}_{21} \quad \mathbf{G} = \frac{\bar{\rho}}{\rho} \mathbf{F}_{11}^{-1} \mathbf{F}_{12} \mathbf{F}_{22}^{-1}. \quad (43)$$

After some manipulation to the additional interference and noise term of the equation (38) $(\mathbb{E}[\mathbf{F}_{R_1}^{-1} \mathbf{\Omega} \mathbf{\Omega}^\dagger \mathbf{F}_{R_1}^{-1}])$, one can show that the instantaneous interference and noise at Relay 1 which is given by

$$\mathbf{\Lambda} = \mathbf{F}_{R_1}^{-1} [\mathbb{E}|x_{1,2}|^2 \mathbf{F}_{11} \mathbf{\Psi} \mathbf{F}_{11}^\dagger + \mathbb{E}|x_{2,1}|^2 \mathbf{F}_{12} \mathbf{\Upsilon} \mathbf{F}_{12}^\dagger + \sigma^2 \mathbf{I}] \mathbf{F}_{R_1}^{-1} \quad (44)$$

where

$$\mathbf{\Psi} = P_{11} \mathbf{C} \mathbf{C}^\dagger + P_{12} \mathbf{D} \mathbf{D}^\dagger + P_{22} \mathbf{F} \mathbf{F}^\dagger + P_{21} \mathbf{G} \mathbf{G}^\dagger \quad (45)$$

$$\mathbf{\Upsilon} = P_{21} \mathbf{A} \mathbf{A}^\dagger + P_{22} \mathbf{B} \mathbf{B}^\dagger. \quad (46)$$

Finally instantaneous SINR is given for the symbol $x_{1,1}$ by

$$\phi_{\text{inst}_{x_{1,1}}} = \frac{\mathbb{E}|x_{1,1}|^2}{\mathbf{\Lambda}(1,1)} \quad (47)$$

where $\mathbf{\Lambda}(1,1)$ is the first entry of the matrix $\mathbf{\Lambda}$ and denotes the additional interference and noise for the symbol $x_{1,1}$.

Examining the first term in $\mathbf{\Lambda}$ which is one of the interference terms due to imperfect CSI and is given by:

$$\mathbf{\Lambda}_1 = \mathbf{F}_{R_1} \mathbb{E}|x_{1,2}|^2 \mathbf{F}_{11} P_{11} \mathbf{C} \mathbf{C}^\dagger \mathbf{F}_{11}^\dagger \mathbf{F}_{R_1}^{-1}, \quad (48)$$

one can show that $\lambda_{11} = \mathbf{\Lambda}_1(1,1)$ is given by

$$\lambda_{11} = \frac{P_{11} \mathbb{E}|x_{1,2}|^2 (\frac{\bar{\rho}}{\rho})^2 \Delta_2}{|\Delta|^2} \quad (49)$$

where

$$\Delta_2 = \left(\frac{1}{|f_{11}(1)|^2} + \frac{1}{|f_{11}(2)|^2} \right) \kappa \quad (50)$$

and $\kappa = |f_{12}(2)|^2 |f_{21}(2)|^2 |f_{12}(1)|^2 |f_{21}(1)|^2$. Additional terms in (48) follow similar structure. Noting that $|\Delta|^2$ is defined in (28) we observe that each of the interference terms will have infinite mean, thus resulting in severe degradation in SINR at the relay.

C. End-To-End Interference with Imperfect CSI

Having derived the SINR at the relay stage, it is straightforward to show that the end-to-end SINR at the destination can be expressed as the sum of the individual interference components arising from each hop. Without loss of generality, we consider $M = 2$. From (11) and (12), we have that the signal at Relay 1 is

$$\mathbf{X}_{R_1} = \begin{bmatrix} x_{1,1} \\ x_{1,2} + x_{2,1} \end{bmatrix} + \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} \quad (51)$$

and similarly at Relay 2

$$\mathbf{X}_{R_2} = \begin{bmatrix} x_{1,1} + x_{2,1} \\ x_{1,2} \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} \quad (52)$$

where δ_1, δ_2 and ζ_1, ζ_2 are the additive interference and noise terms arising from the first hop. Then Relay 1 sends $x_{1,1} + \delta_1$ and $x_{1,2} + x_{2,1} + \delta_2$ along the erroneous relay beamforming vectors $\hat{\mathbf{v}}_{R1,1}$ and $\hat{\mathbf{v}}_{R1,2}$ respectively (see Figure 1b). Similarly Relay 2 sends $x_{1,1} + x_{2,1} + \zeta_1$ along the erroneous relay beamforming vector $\hat{\mathbf{v}}_{R2,1}$. Using expansion for $\hat{\mathbf{G}}_{nj}^{-1}$ similar to (37), erroneous relay beamforming vectors can be derived as:

$$\hat{\mathbf{v}}_{R1,1} = \mathbf{v}_{R1,1} = [1, 1]^T \quad (53)$$

$$\hat{\mathbf{v}}_{R2,1} = -\hat{\mathbf{G}}_{22}^{-1} \hat{\mathbf{G}}_{21} \mathbf{v}_{R1,1} \approx \mathbf{v}_{R2,1} + \epsilon_1 \quad (54)$$

$$\hat{\mathbf{v}}_{R1,2} = \hat{\mathbf{G}}_{11}^{-1} \hat{\mathbf{G}}_{12} \hat{\mathbf{G}}_{22}^{-1} \hat{\mathbf{G}}_{21} \mathbf{v}_{R1,1} \approx \mathbf{v}_{R1,2} + \epsilon_2. \quad (55)$$

where the error components, ϵ_1 and ϵ_2 can be derived replacing \mathbf{F}_{jk} channel coefficients with \mathbf{G}_{jk} channel coefficients in (37). Because we are sending M-1 symbols from the second relay, the second term is not relayed. Simple substitution of (51) into (15) gives

$$\begin{aligned} \hat{\mathbf{Y}}_1 &= \mathbf{G}_{11} [\hat{\mathbf{v}}_{R1,1}(x_{1,1} + \delta_1) + \hat{\mathbf{v}}_{R1,2}(x_{1,2} + x_{2,1} + \delta_2)] \\ &\quad + \mathbf{G}_{12} \hat{\mathbf{v}}_{R2,1}(x_{1,1} + x_{2,1} + \zeta_1) + \mathbf{N}'_1. \end{aligned} \quad (56)$$

where \mathbf{N}'_1 is the additive white Gaussian noise in the second hop. Using (54) and (55),

$$\begin{aligned} \hat{\mathbf{Y}}_1 &= \mathbf{G}_{11} [\mathbf{v}_{R1,1}(x_{1,1} + \delta_1) + \mathbf{v}_{R1,2}(x_{1,2} + x_{2,1} + \delta_2)] \\ &\quad + \mathbf{G}_{12} \mathbf{v}_{R2,1}(x_{1,1} + x_{2,1} + \zeta_1) + \tilde{\mathbf{N}}_1. \end{aligned} \quad (57)$$

where $\tilde{\mathbf{N}}_1$ is the combined noise and interference term which includes interference arising from the error components of beamforming vectors, ϵ_1 and ϵ_2 , in the second hop. Similar analysis can be applied to destination 2. Thus we have shown that, under the approximation in (37), destination receives the symbol $\hat{x}_{1,1}$ corrupted by additive interference and noise due to CSI errors at both stages.

IV. SIMULATION RESULTS

In Figure 2 we plot the CDFs of SNR and SINR for symbol $x_{1,1}$ at Relay 1 using the equations (31) and (47). The figure demonstrates the degradation of SINR due to the increasing ρ , as expected.

In Figure 3, we investigate the CDF of the interference term for the symbol $x_{1,1}$ at Relay 1, $\Lambda(1,1)$, using the equation (44) without noise ($\sigma = 0$). Analytical results agree closely to simulation results with the discrepancy resulting from neglecting $(\frac{\bar{p}}{\rho})^2$ terms which increase with decreasing ρ . Note that the simulation results in Figure 3 also include the effect of higher order terms which are neglected in (37) and therefore (44). This difference between analytical and simulation results is more visible for lower interference and also smaller ρ values. We also showed that the interference is closely modelled by a log normal distribution. Because of the prominent tails of a lognormal distributions, the large instantaneous interference degrades the system performance drastically (See Figure 5).

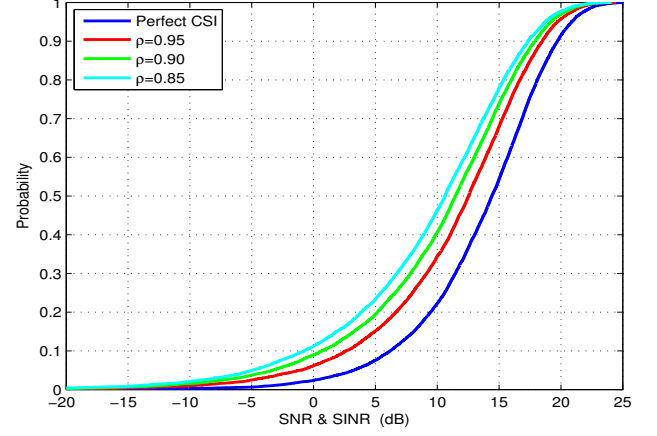


Fig. 2. CDF of SNR for perfect CSI and SINR for imperfect CSI.

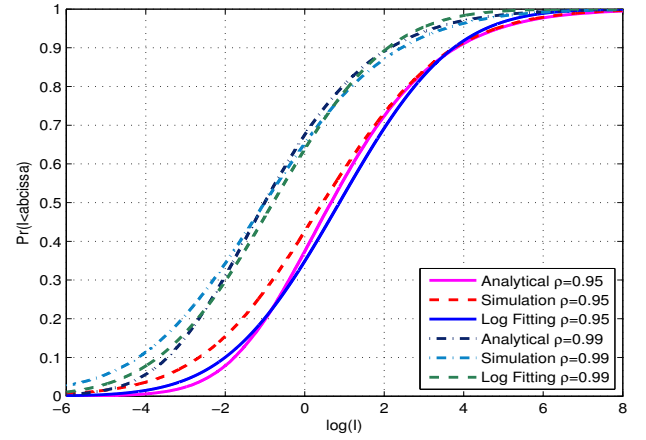


Fig. 3. CDF of interference at Relay 1 for $x_{1,1}$ symbol.

In Figure 4, we simulate the end-to-end interference values as discussed in Section IIIC where we assumed a feedback error while generating both transmitter and relay beamforming vectors. As with the interference at the relay stage, the overall interference is shown to also follow a log normal distribution.

In Figure 5, we investigate the bit error rates of each symbol for $M = 2$ with QPSK modulation. The channel at both hops is assumed to be complex Gaussian and we consider both perfect and imperfect CSI. For the imperfect CSI case $\rho = 0.99$ is chosen. We note that $x_{1,1}$, $x_{1,2}$ and x_2 are corrupted by different noise levels as a result of aligning them into different dimensions during transmission and also due to the decoding technique. For example after decoding x_{11} from the second dimension in order to decode x_{12} , the decision errors for x_{11} impact decoding of x_{12} . It is also seen that bit error rate performances degrades drastically with as little estimation error as $\rho = 0.99$.

Finally, in Figure 6, we investigate symbol error rates for Receiver 1, considering different scenarios for channel

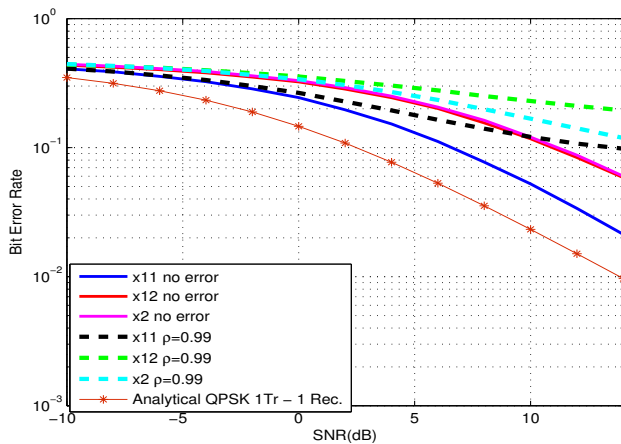
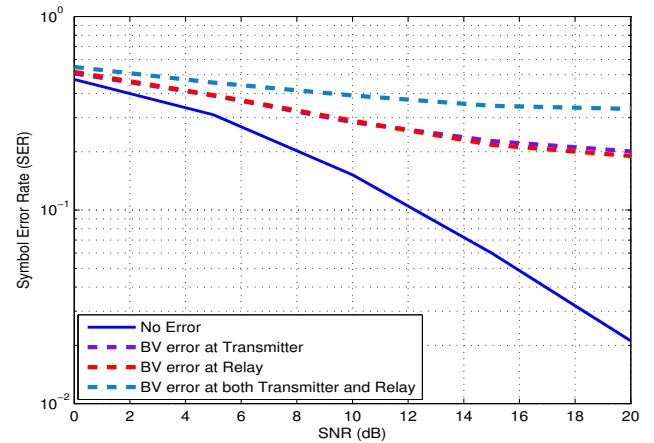
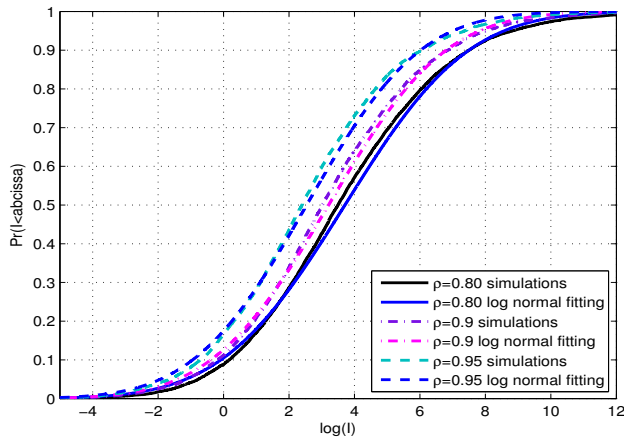


Fig. 5. Bit Error Rates of each symbol with Perfect and Imperfect CSI Cases.

estimation error. ρ is assumed to be 0.99 and we consider 3 cases, error only at transmitter beamforming vectors, error only at relay beamforming vectors and third case is both transmitter and relay beamforming vectors are erroneous. We note that, the system performs worse when beamforming vectors are erroneous only at transmitters, rather than only at relays. On the other hand, the performance in the case of error at both beamforming vectors at transmitters and relays is the worst as expected.

V. CONCLUSION

In this study, we reviewed the aligned interference neutralisation for $2 \times 2 \times 2$ interference channels with imperfect channel state information. We derived analytical expressions for SNR and SINR for perfect and imperfect CSI. We have shown that channel estimation errors introduce a severe degradation. In conclusion, aligned interference neutralisation for multihop networks is very prone to channel estimation errors that accumulate over time. This leads us to conclude that the gain proposed by the $2 \times 2 \times 2$ channel is only achievable in an

ideal setting. The zero forcing receivers also contribute further to noise enhancement. As a future work, multiple antennas can be implemented in sources, relays and destinations in order to exploit new spatial dimensions. It would be also beneficial to investigate in the future if beamforming vectors can be optimized in order to improve the performance with imperfect CSI. Furthermore, different kind of receivers instead of zero forcing can be implemented in order to reduce channel estimation error impact in the system performance.

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